The Transition Curves (Spiral Curves)

The transition curve (spiral) is a curve that has a varying radius. It is used on railroads and most modern highways. It has the following purposes:

1- Provide a gradual transition from the tangent \((r=\infty)\) to a simple circular curve with radius \(R\)
2- Allows for gradual application of superelevation.

Advantages of the spiral curves:
1- Provide a natural easy to follow path such that the lateral force increase and decrease gradually as the vehicle enters and leaves the circular curve
2- The length of the transition curve provides a suitable location for the superelevation runoff.
3- The spiral curve facilitate the transition in width where the travelled way is widened on circular curve.
4- The appearance of the highway is enhanced by the application of spiral curves.

When transition curves are not provided, drivers tend to create their own transition curves by moving laterally within their travel lane and sometimes the adjoining lane, which is risky not only for them but also for other road users.

Minimum length of spiral.

Minimum length of spiral is based on consideration of driver comfort and shifts in the lateral position of vehicles. It is recommended that these two criteria be used together to determine the minimum length of spiral.
Thus, the minimum spiral length can be computed as:

\[
L_{s,\text{min}} = \text{larger of:} \quad L_{s,\text{min}} = \sqrt{\frac{24R}{P_{\text{min}}}} \\
L_{s,\text{min}} = \frac{V^3}{46.7RC}
\]

- \(L_{s,\text{min}}\) = minimum length of spiral, m;
- \(P_{\text{min}}\) = minimum lateral offset (shift) between the tangent and circular curve (0.20 m);
- \(R\) = radius of circular curve, m;
- \(V\) = design speed, km/h;
- \(C\) = maximum rate of change in lateral acceleration (1.2 m/s\(^3\))
Maximum length of spiral. International experience indicates that there is a need to limit the length of spiral transition curves. Safety problems have been found to occur on spiral curves that are long (relative to the length of the circular curve). Such problems occur when the spiral is so long as to mislead the driver about the sharpness of the approaching curve. Maximum length of the spiral can be calculated using:

\[
L_{s,\text{max}} = \sqrt{24R P_{\text{max}}}
\]

where:
- \(L_{s,\text{max}}\) = maximum length of spiral, m;
- \(P_{\text{max}}\) = maximum lateral offset between the tangent and circular curve (1.0 m);
- \(R\) = radius of circular curve, m

Desirable length of spiral

<table>
<thead>
<tr>
<th>design speed km/hr</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
<th>110</th>
<th>120</th>
<th>130</th>
</tr>
</thead>
<tbody>
<tr>
<td>spiral length m</td>
<td>11</td>
<td>17</td>
<td>22</td>
<td>28</td>
<td>33</td>
<td>39</td>
<td>44</td>
<td>50</td>
<td>56</td>
<td>61</td>
<td>67</td>
<td>72</td>
</tr>
</tbody>
</table>
Basic Equations

Equation Of The Spiral:

\[ y = \frac{L_s^3}{6 R L_s} \]

Example: \( R = 500 \text{m}, L_s = 70 \text{m} \) find the tangent offset \((y)\) at distances every 20m from point \( TS \)

<table>
<thead>
<tr>
<th>( l_s ) (m)</th>
<th>( y ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
</tr>
</tbody>
</table>

\[ \alpha (rad) = \frac{L_s}{2 R} \]

the tangent \((I - TS)\) = \( \frac{L_s}{2} + (R + P) \tan \frac{\Delta}{2} \)

deflection angle of the spiral \((rad)\) = \( \frac{l_s^2}{6 R L_s} \)
Example: in order to improve an existing circular curve $R=250m$, $\Delta=31^\circ 20$. Determine minimum and maximum length of the spiral curve. The design speed $=80km/hr$

$$L_{s,\text{min}} = \sqrt{24R P_{\text{min}}} \quad L_{s,\text{min}} = \sqrt{24 \times 250 \times 0.2} = 34.64m$$

$$L_{s,\text{min}} = \frac{V^3}{46.7 \times R \times C} \quad L_{s,\text{min}} = \frac{80^3}{46.7 \times 250 \times 1.2} = 36.5m$$

Use $L_{s,\text{min}} = 36.5m$

$$L_{s,\text{max}} = \sqrt{24R P_{\text{max}}} = \sqrt{24 \times 250 \times 1} = 77.45m$$

Therefore $36.5m < L_s < 77.5m$

Example: two tangents intersect at a deflection angle $= 35^\circ$ at station I= 23+20. These tangents are to be connected by two similar transition curves 75m long and a circular curve, $R=300m$ . calculate stations TS,SC,CS,ST

$$\alpha \text{ central angle of the spiral (rad)} = \frac{L_s}{2R} = \frac{75}{2 \times 300} = 0.125 \text{ rad} = 7^\circ 09'$$

$35-2\alpha$ (central angle of the circular curve)$= 20^\circ 42'$

Length of the circular curve $L= 20^\circ 42' \times \frac{\pi}{180} \times 300 = 108.38$

Shift$= \frac{L_s^2}{24R} = \frac{75^2}{24 \times 300} = 0.78m$

Tangent length$= \frac{75}{2} + (300 + 0.78) \tan \frac{35^\circ}{2} = 132.33m$

Station TS = station I- tangent $= 21+87.67$

Station SC= station TS $+ L_s = 22+62.67$

Station CS= station SC$ + L_{\text{circular}} = 23+71.10$

Station ST = station CS$ + L_s = 24+46.05$

Setting out of a transition curve(deflection angle method)
Example: deflection angle between tangents = 30°, station TS = 7+08, radius of circular curve $R = 382$m, length of the transition curve = 120m. Prepare a setting out table to set out the transition and circular curve at points every 20m.

$$\alpha \ (rad) = \frac{L_s}{2R} = \frac{120}{2 \times 382} = 0.15706 \ rad = 9°$$

$$L_{circular} = (30 - 2 \times 9) \times \frac{\pi}{180} \times 382 = 80m$$

Station SC = 708 + 120 = 8 + 28

Station CS = 828 + 80 = 9 + 08

Station ST = 908 + 120 = 10 + 28

**deflection angle of the spiral (rad) =** $\frac{l_s^2}{6RL_s}$ ( $l_s$ is cumulative value i.e 20, 40, 60,..)

1- Setting out the spiral

Theodolite at station TS. Zero horizontal angle towards point I

<table>
<thead>
<tr>
<th>station</th>
<th>chord length (m)</th>
<th>curve length $l_s$</th>
<th>deflection angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>7+08</td>
<td>TS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7+20</td>
<td>12</td>
<td>12</td>
<td>00° 01´ 50˝</td>
</tr>
<tr>
<td>7+40</td>
<td>20</td>
<td>32</td>
<td>00° 12´ 50˝</td>
</tr>
<tr>
<td>7+60</td>
<td>20</td>
<td>52</td>
<td>01° 33´ 50˝</td>
</tr>
<tr>
<td>7+80</td>
<td>20</td>
<td>72</td>
<td>01° 04 50</td>
</tr>
<tr>
<td>8+00</td>
<td>20</td>
<td>92</td>
<td>01° 45 50</td>
</tr>
<tr>
<td>8+20</td>
<td>20</td>
<td>112</td>
<td>02° 36 50</td>
</tr>
<tr>
<td>8+28</td>
<td>SC</td>
<td>120</td>
<td>03° 00 00 = $\alpha/3$</td>
</tr>
</tbody>
</table>

$\text{Max. deflection angle} = 3°$

$L_s = 120m$
2- Setting out of the circular curve

Theodolite at SC. Sight TS make reading \(=360- (\alpha - \text{max def. angle})= 360- (9-3)\)
\[=354^\circ\]

Transit the theodolite and set out as below

<table>
<thead>
<tr>
<th>station</th>
<th>arc length</th>
<th>central angle/2</th>
<th>tangential angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>8+28 SC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8+40</td>
<td>12</td>
<td>00 54</td>
<td>00° 54´</td>
</tr>
<tr>
<td>8+60</td>
<td>20</td>
<td>01 30</td>
<td>02° 24´</td>
</tr>
<tr>
<td>8+80</td>
<td>20</td>
<td>01 30</td>
<td>03 54</td>
</tr>
<tr>
<td>9+00</td>
<td>20</td>
<td>01 30</td>
<td>05 24</td>
</tr>
<tr>
<td>9+08</td>
<td>08</td>
<td>00 36</td>
<td>06 00</td>
</tr>
</tbody>
</table>